Christ Church Grammar School

2019
TEST 3

MATHEMATICS SPECIALIST Year 12

Section One:
Calculator-free

Your name SOLUTIONS

Teacher's name _____

Time and marks available for this section

Reading time for this section:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 **(7 marks)**

(a) Determine the parametric equations of the line through the points $A(-1, 2, 3)$ and $B(2, 0, -3)$. (3 marks)

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ -2 \\ -6 \end{pmatrix}$ $\quad r = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ -6 \end{pmatrix}$ ✓ writing vector equation with correct direction can also have $\overrightarrow{BA}$

$x = -1 + 3\lambda$

$y = 2 - 2\lambda$ ✓✓ writing parametric equations. 1 mark, 1 correct 2 marks, 3 correct

$z = 3 - 6\lambda$

(b) Hence find where the line meets the plane with equation $x - 2y + 3z = 26$. (4 marks)

subst into plane $x - 2y + 3z = 26$

$-1 + 3\lambda - 2(2 - 2\lambda) + 3(3 - 6\lambda) = 26$ ✓ substs into plane

$-1 + 3\lambda - 4 + 4\lambda + 9 - 18\lambda = 26$

$-11\lambda = 22$

$\lambda = -2$ ✓ simplifies & solves for λ

$\therefore$ when $\lambda = -2$

$x = -1 + -6$

$= -7$

$y = 2 + 4$

$= 6$ ✓ find each co-ordinate

$z = 3 + 12$

$= 15$

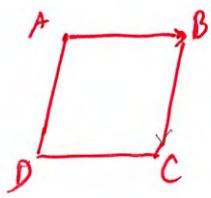
$\therefore$ intersects at $(-7, 6, 15)$ ✓ writes ~~at~~ point of intersection

See next page

Question 2 **(8 marks)**

$A(2,4,2)$, $B(-1,2,3)$, $C(-3,3,6)$ and $D(0,5,5)$ are vertices of a quadrilateral.

(a) Prove that ABCD is a parallelogram. (3 marks)



$\overrightarrow{AB} = \begin{pmatrix} -3 \\ -2 \\ 1 \end{pmatrix}$ $\overrightarrow{DC} = \begin{pmatrix} -3 \\ -2 \\ 1 \end{pmatrix}$ parallel as same dirn ✓

Finds direction of 2 opposite sides

$\overrightarrow{BC} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$ $\overrightarrow{AD} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$ parallel as same dirn ✓

Finds direction of other 2 opposite sides ✓

opposites are parallel ∴ shape is a parallelogram ✓

Final statement.

(b) Calculate $|\overrightarrow{AB}|$ and $|\overrightarrow{BC}|$. What can be said about ABCD? (2 marks)

$|\overrightarrow{AB}| = \sqrt{(-3)^2+(-2)^2+1^2}$ $|\overrightarrow{BC}| = \sqrt{(-2)^2+1^2+3^2}$

$= \sqrt{14}$ $= \sqrt{14}$ ✓

For both correct calculations

∴ ABCD is a rhombus ✓ For statement as all sides the same length.

(c) Calculate $\overrightarrow{AC} \cdot \overrightarrow{BD}$. What property of figure ABCD can be found to be valid? (3 marks)

$\overrightarrow{AC} \cdot \overrightarrow{BD}$

$\begin{pmatrix} -5 \\ -1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ ✓ correct calculation of $\overrightarrow{AC}$ and $\overrightarrow{BD}$

$= -5 - 3 + 8$

$= 0$ ✓ correct calculation of dot product (from their values of $\overrightarrow{AC}$ & $\overrightarrow{BD}$).

∴ diagonals are perpendicular. ✓

Final statement.

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

Christ Church Grammar School

2019
TEST 3

MATHEMATICS SPECIALIST Year 12

Section Two:
Calculator-assumed

Your name SOLUTIONS

Teacher's name _____

Time and marks available for this section

Reading time for this section:	3 minutes
Working time for this section:	30 minutes
Marks available:	30 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates and up to three calculators approved for use in the WACE examinations

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See next page

Question 3 **(5 marks)**

A sphere has a centre $(2, 3, 4)$ and radius 6. Find the

(a) Cartesian equation of the sphere. (1 mark)

$(x-2)^2 + (y-3)^2 + (z-4)^2 = 36$ ✓

(b) Hence or otherwise, find the intersection of the sphere with the line $\boldsymbol{r} = 2\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k} + t(\boldsymbol{i} + \boldsymbol{j}), t \in \mathbb{R}$. (4 marks)

$\underset{\sim}{r} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $\quad x = 2+t$, $y = 3+t$, $z = 4$ ✓ Find parametric equations

sub parametric into sphere

$(2+t-2)^2 + (3+t-3)^2 + (4-4)^2 = 36$ ✓ sub into sphere

Note: mark land 2 can be implied here

$2t^2 = 36$

$t = \pm\sqrt{18}$

$= \pm 3\sqrt{2}$ ✓ solve for t

∴ Intersects at $(2-3\sqrt{2},\ 3-3\sqrt{2},\ 4)$

and $(2+3\sqrt{2},\ 3+3\sqrt{2},\ 4)$ ✓ Find intersection points

or accept $(2-3\sqrt{2})\underset{\sim}{i} + (3-3\sqrt{2})\underset{\sim}{j} + 4\underset{\sim}{k}$

and $(2+3\sqrt{2})\underset{\sim}{i} + (3+3\sqrt{2})\underset{\sim}{j} + 4\underset{\sim}{k}$

Note: $\frac{2}{4}$ if only $+3\sqrt{2}$ as a solution.

See next page

Question 4 **(8 marks)**

(a) Show that the lines with equations

$l_1: \frac{x-4}{3} = \frac{y+1}{-2} = \frac{z-2}{1}$ and $l_2: \frac{x-5}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect and hence find the point of intersection. (5 marks)

$L_1: x = 3t + 4$, $y = -2t - 1$, $z = t + 2$

$L_2: x = 2s + 5$, $y = -s - 1$, $z = s + 3$ ✓ Find eqns for L_1 + L_2

$x = x \quad 3t + 4 = 2s + 5$ ①

$y = y \quad -2t - 1 = -s - 1$ ②

$z = z \quad t + 2 = s + 3$ ③ ✓

let $x = x$, $y = y$, $z = z$

→ solve $t = -1$, $s = -2$ ✓ solve for variables

check in ① (as used ② + ③)

$(3(-1) + 4) = 2(-2) + 5$

$1 = 1$ ✓ ✓ check in 3rd equation (must do for 4th mark)

∴ P.O.I $(1, 1, 1)$ ✓ Find P.O.I.

(b) Find the vector equation, in normal form, of the plane containing l_1 and l_2. (3 marks)

$\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$ ✓

correct value of cross product

$\underset{\sim}{r} \cdot \underset{\sim}{n} = c$

$\underset{\sim}{r} \cdot \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$

$\underset{\sim}{r} \cdot \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = -1$ ✓ correct value of dot product ✓

vector equation in normal form.

Note: can get 2/3 if cross product is incorrect.

See next page

Question 5 **(5 marks)**

Two particles, A and B, have initial positions of $2\boldsymbol{i} + 4\boldsymbol{j}$ and $28\boldsymbol{i} - 4\boldsymbol{j}$ respectively.

If the velocity of A is $3\boldsymbol{i} - 2\boldsymbol{j}$ and the velocity of B is $-2\boldsymbol{i} - \boldsymbol{j}$, determine the point of intersection of their paths and state whether or not the particles meet at this point.

$\underset{\sim}{r}_A = \begin{pmatrix} 2+3\lambda \\ 4-2\lambda \end{pmatrix}$ $\underset{\sim}{r}_B = \begin{pmatrix} 28-2\mu \\ -4-\mu \end{pmatrix}$ ✓ Finds vector equations of both lines - unknowns must be different

or $\underset{\sim}{r}_A = 2\underset{\sim}{i} + 4\underset{\sim}{j} + \lambda(3\underset{\sim}{i} - 2\underset{\sim}{j})$ $\underset{\sim}{r}_B = 28\underset{\sim}{i} - 4\underset{\sim}{j} + \mu(-2\underset{\sim}{i} - \underset{\sim}{j})$

$x = x$ $2 + 3\lambda = 28 - 2\mu$

$y = x$ $4 - 2\lambda = -4 - \mu$. ✓ let $x = x$, $y = y$

Find values of unknowns

$\therefore \lambda = 6$

$\mu = 4$ ✓

$\therefore$ the particles do not meet at the point. ✓ makes statement

p.oint of intersection of their paths is $(20, -8)$ ✓

ie $\lambda = 6$ $\underset{\sim}{r}_A = \begin{pmatrix} 2+18 \\ 4-12 \end{pmatrix}$

Finds p.o.i of paths.

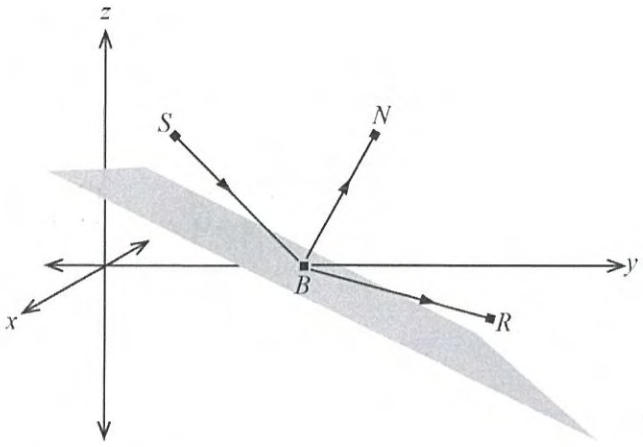
or $\mu = 4$ $\underset{\sim}{r}_B = \begin{pmatrix} 28-8 \\ -4-4 \end{pmatrix}$

See next page

Question 6 **(7 marks)**

A laser pointer at point S directs a highly focused beam of light towards a mirror. The beam bounces off the mirror at point B and is then reflected away from the mirror toward point R.

The mirror's surface is given by the equation $\boldsymbol{r} \cdot (\boldsymbol{j} + 2\boldsymbol{k}) = 9$ and the laser pointer is positioned at point S with position vector $-2\boldsymbol{i} + 3\boldsymbol{j} + 6\boldsymbol{k}$. The laser pointer is held so that the beam is pointed in the direction $\boldsymbol{d}_1 = \boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$.



(a) Determine the position vector for point B. (4 marks)

$\overrightarrow{SB} \Rightarrow \underset{\sim}{r} = \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ ✓ Finds equation of line SB

intersects with beam, subst $\overrightarrow{SB}$ into mirror surface

$\begin{pmatrix} -2+\lambda \\ 3+\lambda \\ 6-\lambda \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = 9$ ✓ subst into mirror surface

$3 + \lambda + 12 - 2\lambda = 9$

$\lambda = 6$ ✓ solves for λ (or unknown)

subst $\lambda = 6$ into $\overrightarrow{SB}$

$\therefore \overrightarrow{OB} = \begin{pmatrix} 4 \\ 9 \\ 0 \end{pmatrix}$ or $4\underset{\sim}{i} + 9\underset{\sim}{j}$ ✓ Finds position vector. must be a vector not (4, 9, 0) co-ordinate.

See next page

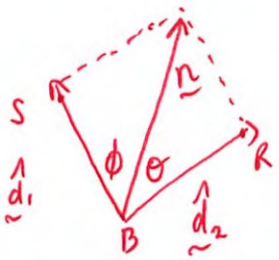
Question 6 continued

The laser beam is reflected away from the mirror so that:

- the angle of the incoming beam $\overrightarrow{SB}$ to the normal of the mirror is equal to the angle of the reflected beam $\overrightarrow{BR}$ to the normal of the mirror i.e. $s\angle SBN = s\angle NBR$
- the incoming beam $\overrightarrow{SB}$, the normal of the mirror and the reflected beam $\overrightarrow{BR}$ are all contained in one plane.

Let $\widehat{d_2}$ = the unit vector in the direction of the reflected beam $\overrightarrow{BR}$ i.e. $|\widehat{d_2}| = 1$.

(b) Determine the unit vector $\widehat{d_2}$ giving components correct to 0.01. (3 marks)



normal vector will bisect ∠SBR

unit vector $\hat{d}_1 = \frac{1}{\sqrt{3}}\begin{pmatrix}-1\\-1\\1\end{pmatrix}$

using parallelogram

$\hat{d}_1 + \hat{d}_2 = k\underset{\sim}{n}$ ✓ makes statements re unit vectors.

$\hat{d}_2 = k\begin{pmatrix}0\\1\\2\end{pmatrix} - \frac{1}{\sqrt{3}}\begin{pmatrix}-1\\-1\\1\end{pmatrix}$

$= \begin{pmatrix}\frac{1}{\sqrt{3}}\\ k+\frac{1}{\sqrt{3}}\\ 2k-\frac{1}{\sqrt{3}}\end{pmatrix}$

$\overrightarrow{SB} = \begin{pmatrix}1\\1\\-1\end{pmatrix}$

$\therefore \overrightarrow{BS} = \begin{pmatrix}-1\\-1\\1\end{pmatrix}$

also accept

$\overrightarrow{SB} = \begin{pmatrix}6\\6\\-6\end{pmatrix}$

$\therefore \overrightarrow{BS} = \begin{pmatrix}-6\\-6\\6\end{pmatrix}$

if not ÷6.

$|\hat{d}_2| = 1$

So $1 = \left(\frac{1}{\sqrt{3}}\right)^2 + \left(k+\frac{1}{\sqrt{3}}\right)^2 + \left(2k-\frac{1}{\sqrt{3}}\right)^2$ ✓ sets equation = 1

$\therefore k = \frac{2}{5\sqrt{3}}$ or 0, cannot be 0

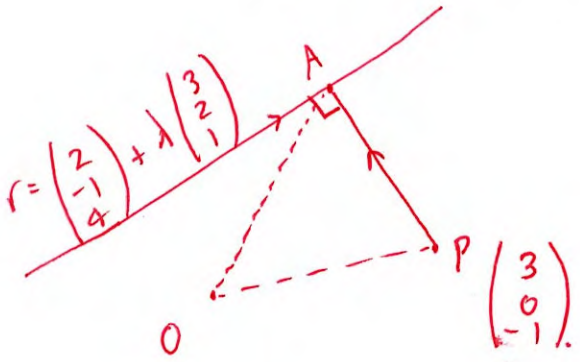
$k = \frac{2}{5\sqrt{3}}$ $\left(\text{or } \frac{2\sqrt{3}}{15}\right)$

hence $\hat{d}_2 = \begin{pmatrix}\frac{1}{\sqrt{3}}\\ \frac{7}{5\sqrt{3}}\\ -\frac{1}{5\sqrt{3}}\end{pmatrix} = \begin{pmatrix}0.58\\0.81\\-0.12\end{pmatrix}$ ✓ solves for $\hat{d}_2$ with rounding

See next page

Question 7 **(5 marks)**

Find the shortest distance from the point $(3, 0, -1)$ to the line with equation $\boldsymbol{r} = 2\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k} + \lambda(3\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k})$.



$\overrightarrow{PA} = \overrightarrow{PO} + \overrightarrow{OA}$

$= \begin{pmatrix}-3\\0\\1\end{pmatrix} + \begin{pmatrix}2+3\lambda\\-1+2\lambda\\4+\lambda\end{pmatrix}$

$= \begin{pmatrix}-1+3\lambda\\-1+2\lambda\\5+\lambda\end{pmatrix}$ ✓

calculates $\overrightarrow{PA}$ (or $\overrightarrow{AP}$)

$\overrightarrow{PA} \cdot \begin{pmatrix}3\\2\\1\end{pmatrix} = 0$ ✓ statement $\overrightarrow{PA}$. dir^n $= 0$

$\begin{pmatrix}-1+3\lambda\\-1+2\lambda\\5+\lambda\end{pmatrix} \cdot \begin{pmatrix}3\\2\\1\end{pmatrix} = 0$

$-3+9\lambda-2+4\lambda+5+\lambda = 0$

$\lambda = 0$ ✓ solves for λ (or unknown)

$\overrightarrow{PA} = \begin{pmatrix}-1\\-1\\5\end{pmatrix}$ ✓ calculates closest point

$\therefore |\overrightarrow{PA}| = \sqrt{(-1)^2 + (-1)^2 + 5^2}$

$= \sqrt{27}$

$= 3\sqrt{3}$ ✓ calculates closest distance

$(5.19615 \simeq 5.2)$

End of Questions

Additional working space

Question number: _____

Additional working space

Question number: _____